

Question 11 (13 marks)

The family of functions known as *hyperbolic* functions are expressed using the following notation:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Figure 13 shows the graph of $y = \sinh x$.

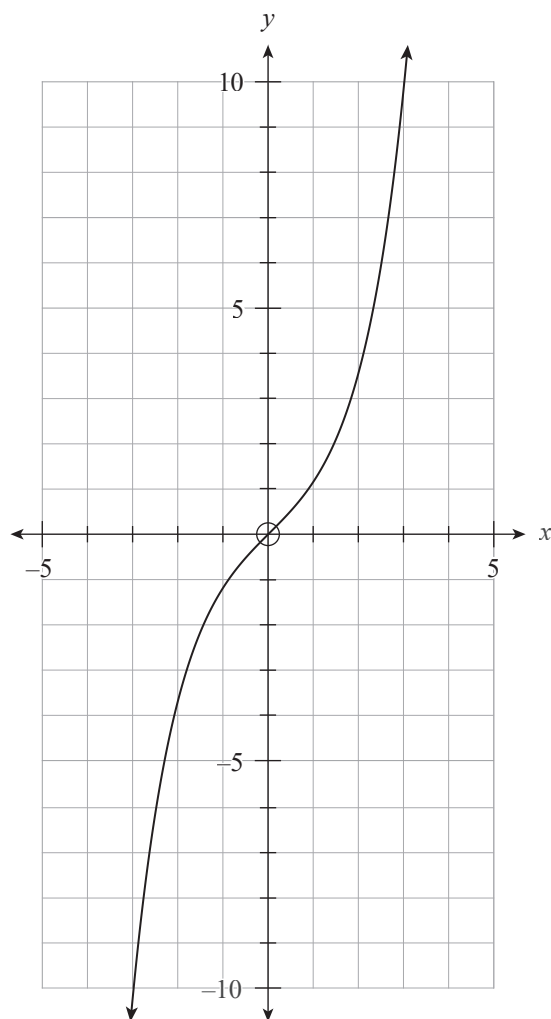


Figure 13

(a) On Figure 13, add a sketch of the graph of $y = \cosh x$.

(2 marks)

One reason for expressing *hyperbolic* functions using the notation on page 14 is that their derivatives follow a similar (*but not identical*) pattern to trigonometric functions. Two examples of the relationship between *hyperbolic* functions and their derivatives are shown in Table 2.

Table 2

<i>Relationship 1</i>	<i>Relationship 2</i>
If $y = \sinh x$, $\frac{dy}{dx} = \cosh x$	If $y = \cosh x$, $\frac{dy}{dx} = \sinh x$

(b) A third relationship is shown in Table 3.

Table 3

<i>Relationship 3</i>
If $y = \tanh x$, $\frac{dy}{dx} = \frac{1}{(\cosh x)^2}$

Show that Relationship 3 is valid.

You may assume that Relationship 1 and Relationship 2 are valid when constructing your answer, if required.

(3 marks)

Recall that *hyperbolic* functions are expressed using the following notation:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Additionally, recall the following relationships in Table 4:

Table 4

<i>Relationship 1</i>	<i>Relationship 2</i>	<i>Relationship 3</i>
If $y = \sinh x$, $\frac{dy}{dx} = \cosh x$	If $y = \cosh x$, $\frac{dy}{dx} = \sinh x$	If $y = \tanh x$, $\frac{dy}{dx} = \frac{1}{(\cosh x)^2}$

(c) Consider the function $f(x) = 13 \tanh x + 10 \ln(\cosh x)$.

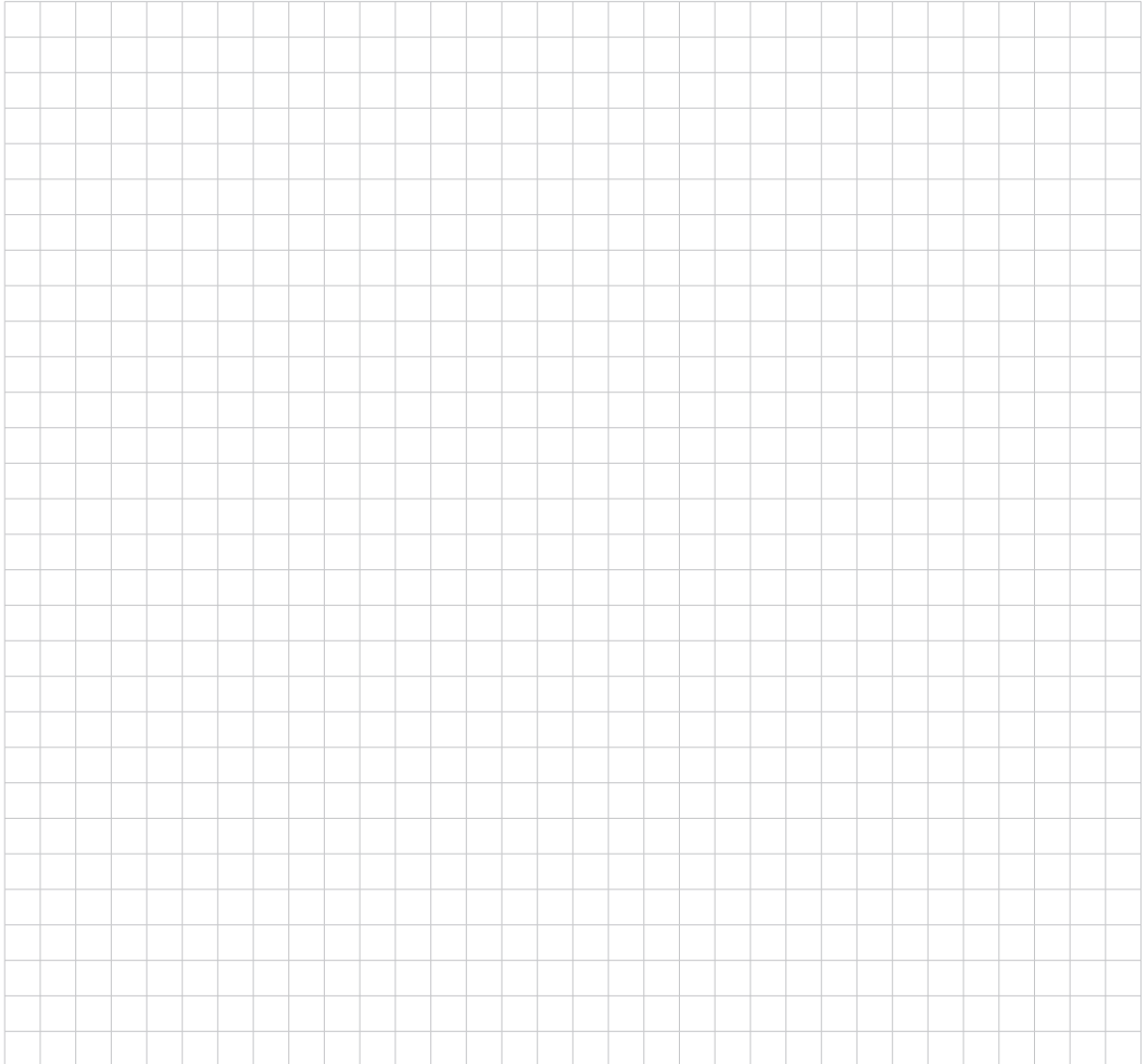
(i) Show that $f''(x) = \frac{10 \cosh x - 26 \sinh x}{(\cosh x)^3}$.

(4 marks)

- (ii) The function $f(x)$ has one point of inflection.

Using an algebraic approach and your answer to part (c)(i), determine the *exact* x -coordinate of the point of inflection.

Express your answer in the form $x = \ln\left(\frac{a}{b}\right)$, where a and b are integers.



(4 marks)