

Question 10

(12 marks)

Consider the function $f(x) = 100 \ln(2x - \sin 2x)$, where $x > 0$ and $f'(x) \geq 0$ for all $x > 0$.

Let A_k represent the k^{th} closest stationary point of $f(x)$ to the origin, where k is a positive integer.

The graph of $y = f(x)$ is shown in Figure 11 with the three closest stationary points to the origin labelled A_1 , A_2 , and A_3 .

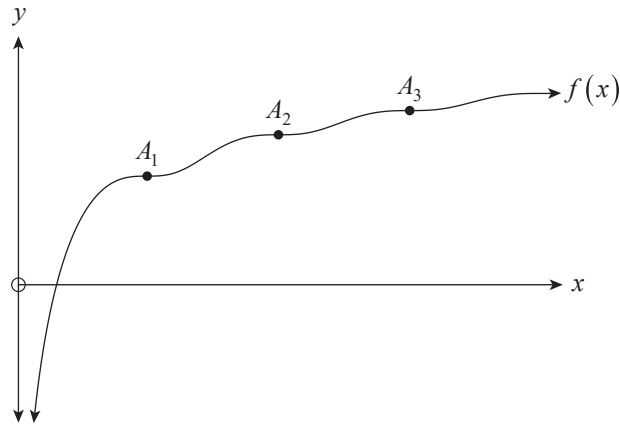


Figure 11

(a) Determine $f'(x)$.

(2 marks)

(b) Using an algebraic approach, show that the *exact* x -coordinate of A_k is $x = \pi k$.

(2 marks)

- (c) Figure 12 shows a section of the graph of the function $y = f(x)$, where $f(x) = 100 \ln(2x - \sin 2x)$ located around consecutive stationary points A_k and A_{k+1} for some value of k .

The labelled distances between A_k and A_{k+1} are:

- d_k , the shortest distance
- v_k , the vertical distance
- h_k , the horizontal distance.

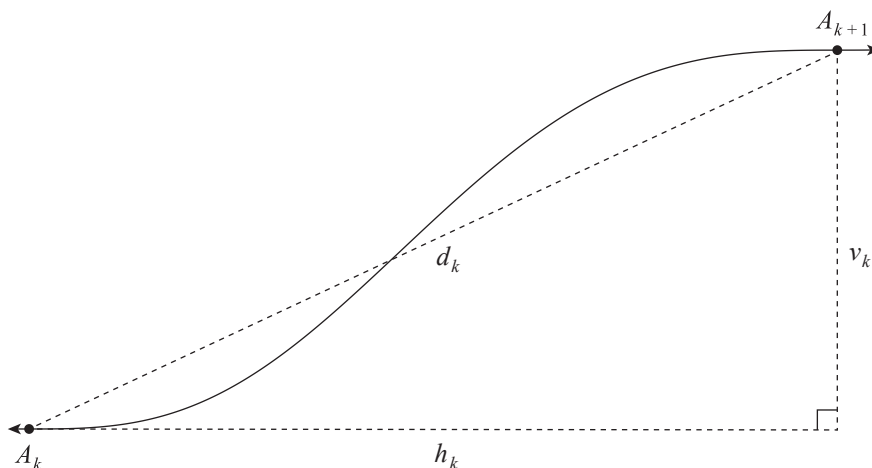


Figure 12

The following conjecture is made:

The vertical distance between the consecutive stationary points A_k and A_{k+1} is

$$v_k = 100 \ln\left(\frac{k+1}{k}\right).$$

- (i) Prove the given conjecture.



(4 marks)

(ii) (1) Hence, determine a formula for d_k , in terms of π and the variable k .

[illegible]

(1 mark)

(2) Hence, determine the smallest value of k such that $d_k < 3.50$.

Give evidence to support your answer.

(3 marks)