

Question 8

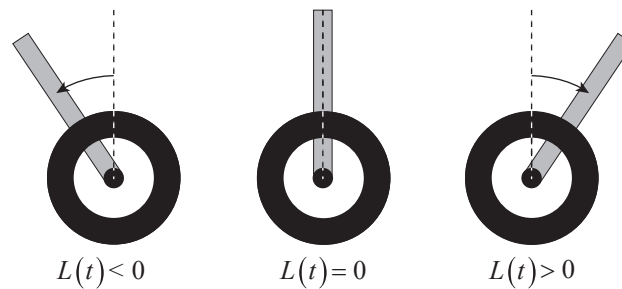
(10 marks)

A self-balancing robot uses a device called a PID controller to stop itself from falling over after being placed on its wheels and released. Let $L(t)$ represent the angle of lean of the robot, at t seconds after release, where $L(t)$ is measured in **radians**. Side-on diagrams of this robot in Figure 6 show various angles of lean, where:

- negative values of $L(t)$ mean the robot has an angle of lean to the left
- $L(t) = 0$ means the robot is vertical
- positive values of $L(t)$ mean the robot has an angle of lean to the right.

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Source: www.superdroidrobots.com

**Figure 6**

The PID controller's settings can be adjusted to change how long it takes the robot to reach an approximately stable vertical position after initially being placed on its wheels and released.

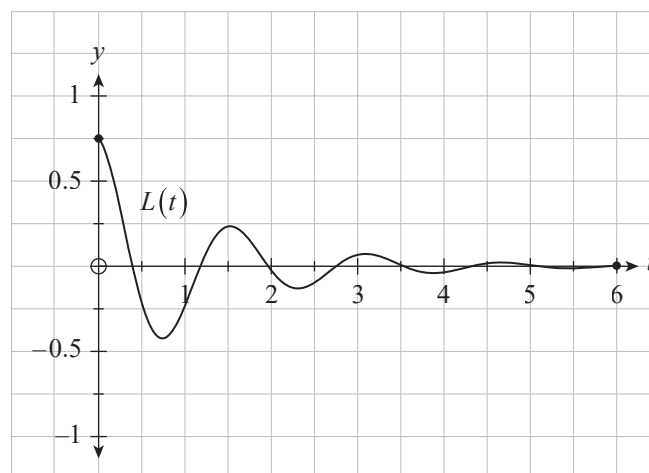
Engineers conducted an experiment using different settings on the PID controller, starting with Setting A.

- (a) Using Setting A, the robot was initially placed on its wheels and released. The resulting angle of lean was recorded for 6 seconds.

Using this setting, the angle of lean of the robot, $L(t)$, at t seconds after release, can be modelled by the function

$$L(t) = 0.75e^{-0.75t} \cos(4t), \text{ where } 0 \leq t \leq 6.$$

A graph of $y = L(t)$ for Setting A is shown in Figure 7.

**Figure 7**

- (i) State the initial angle of lean of the robot, according to the model.

(1 mark)

- (ii) Determine the value of $L'(1.3)$.

(1 mark)

- (iii) Interpret your answer to part (a)(ii) in the context of the problem, stating the appropriate units.

(2 marks)

- (iv) Calculate how long after being released the robot is leaning to the left with its maximum angle, according to the model.

(1 mark)

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- Using Setting B, the angle of lean of the robot, $L(t)$, at t seconds after release, can now be modelled by the new function

A graph of $y = L(t)$ for Setting B is shown in Figure 8.



(ii) Hence, for Setting B, use an algebraic approach to determine the exact values of t such that $L'(t) = 0$, during the first 6 seconds after release.

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