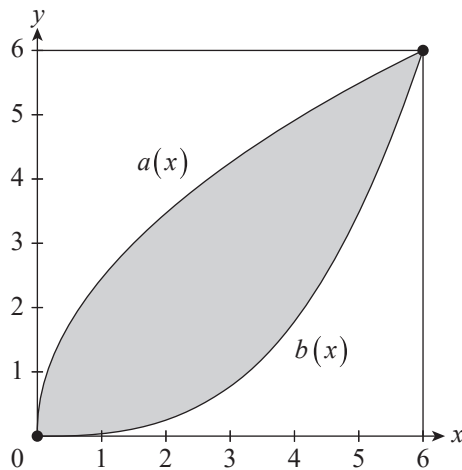


Question 9

(16 marks)

Figure 10 shows a shaded region that is bounded by the graphs of the functions $y = a(x)$ and $y = b(x)$. These functions intersect at the points $(0, 0)$ and $(6, 6)$. The shaded region is contained within the square formed by the x -axis, the y -axis, $x = 6$, and $y = 6$.

**Figure 10**

A Monte Carlo experiment can be used to estimate the proportion of the square that is shaded, and hence to estimate the area of the shaded region. The experiment uses the following steps:

1. independently randomly generate a sample of n points within the square containing the shaded region; each point within the square has an equal probability of being generated
2. count the number of these points, x , that are within the shaded region
3. calculate $\hat{p} = \frac{x}{n}$, the proportion of the sample of points that is within the shaded region.

The proportion of the sample of points that is within the shaded region can be used to estimate A , the area of the shaded region, using

$$\hat{p} \times \text{area of the square.}$$

- (a) A Monte Carlo experiment is conducted using three independently randomly generated points. The results are shown in Figure 11.

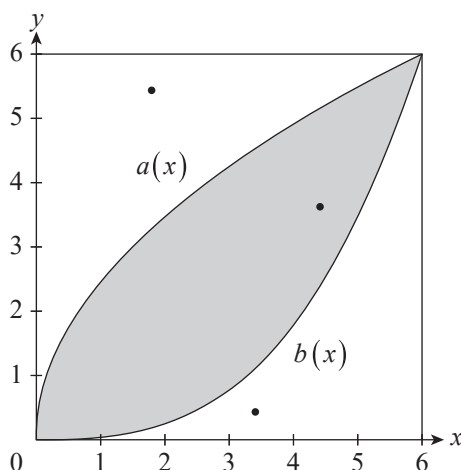


Figure 11

- (i) Calculate $\hat{p}$, the proportion of the sample of points that is within the shaded region.

(1 mark)

- (ii) Hence show that the results from this Monte Carlo experiment indicate that A is estimated to be 12 square units.

(1 mark)

- (iii) State why it may be inappropriate to construct a confidence interval for the proportion of the square that is shaded when using a Monte Carlo experiment involving three independently randomly generated points.

(1 mark)

It is established that the shaded region shown in Figure 10 on page 8 is bounded by $a(x) = (6x)^{\frac{1}{2}}$ and $b(x) = \frac{x^3}{36}$.

- (c) (i) Write an integral expression that can be used to find A , the area of the shaded region that is bounded by $a(x)$ and $b(x)$.

(2 marks)

- (ii) Hence using an algebraic process, determine A .

(3 marks)